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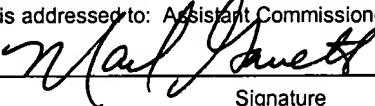
for

Statistical Methods for Analyzing Biological Sequences

by

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1

BACKGROUND OF THE INVENTION

2

3 This application claims priority to provisional patent application Serial
4 No. 60/157,974 filed October 6, 1999, entitled "Statistical Method for Measuring the
5 Energetic Properties of Proteins Through Sequence Analysis." The entire text of the
6 above-referenced disclosure is specifically incorporated by reference herein without
7 disclaimer.

8

9 The Appendix to this specification contains computer-program source code that is
10 the property of the assignee. Copies of the source code may be made as part of making
11 facsimile reproductions of this specification, but all other rights in the source code are
12 reserved. Those with skill in the art having the benefit of this disclosure will understand
13 that the appended source code may be modified as necessary for use with operating
14 systems other than the standard, UNIX-based operating system for which it is currently
15 written. For example, the appended source code may be modified for use with any
16 Microsoft Windows operating system.

17

18 **1. Field of the Invention**

19 The invention relates generally to analyzing biological sequences. This invention
20 relates more particularly to methods for analyzing biological sequences using algorithms,
21 which sequences include, but are not limited to, proteins, ribonucleic acids (RNA),
22 deoxyribonucleic acids (DNA), lipids, and polysaccharides (sugars).

23

24 **2. Description of Related Art**

25 The ability of all cells to recognize their environment and to make appropriate
26 responses to stimuli depends on the organized activity of networks of proteins that we
27 conventionally refer to as the cellular signal transduction machinery. These protein
28 networks show remarkable signal processing properties such as the ability to extract
29 small signals from noise and to adjust their sensitivity to changes in background
30 stimulation while preserving excellent specificity. As used herein, "specificity" is the
31 ability of proteins or protein networks to selectively respond to one stimulus in the

1 background of other potentially competing stimuli. Defects in signaling proteins are
2 commonly the basis for many human diseases, highlighting the need for a fundamental
3 understanding of the mechanisms of signal recognition and processing.

4

5 The basic paradigm of signaling involves the sequential establishment of
6 molecular interactions and the allosteric control of enzyme activities. At an atomic level,
7 these processes reduce to the orderly flow of energy within and between proteins whose
8 structural basis is not generally well understood. For example, the effect of ligand
9 binding at extracellular sites in a transmembrane receptor molecule presumably
10 propagates via the motion of coupled structural elements to induce functional changes in
11 intracellular domains and the subsequent interaction with downstream target proteins.
12 The interaction of one protein with another can be thought of as an energetic perturbation
13 to each binding surface that propagates through the three-dimensional structure to cause
14 specific changes in protein function (Holt, J.M. and Ackers, G.K., *Faseb J.* 9: 210-218,
15 1995; Monod, J. et al., *J. Mol. Biol.* 12: 88-118, 1965; Perry, K.M. et al., *Biochem.* 28:
16 7961-7968, 1989; Pettigrew, D.W. et al., *Proc. Natl. Acad. Sci. U.S.A.* 79: 1849-1853,
17 1982; LiCata, V.J. and Ackers, G.K., *Biochemistry* 34: 3133-3139, 1995; Turner, G.J. et
18 al., *Proteins* 14: 333-350, 1992). The structural basis of this energy propagation is
19 largely unknown, but is likely to be critical in understanding the relationship between
20 protein function and structure.

21

22 At specific protein-protein interfaces, large-scale mutagenesis together with
23 structure determination has begun to define some features of energy parsing. (As used
24 herein, “energy parsing” describes the way that energy is parceled out amongst the
25 amino-acid residues at a particular protein-protein interface. Mutagenesis is a method of
26 generating DNA-level changes to a gene encoding a protein in order to change the
27 identity of an amino acid at a chosen position on the protein.) For example, studies of the
28 interaction of human growth hormone with its receptor show that binding energy is not
29 smoothly distributed over the interaction surface; instead, a few residues comprising only
30 a small fraction of the interaction surface account for the majority of the free energy
31 change (Atwell, S. et al., *Science* 278: 1125-1128, 1997; Clackson, T. and Wells, J.A.,

1 *Science* 267: 383-386, 1995; Wells, J.A., *Proc. Natl. Acad. Sci. U.S.A.* 93: 1-6, 1996; J.
2 A. Wells, *Biotechnol.* 13: 647-651, 1995).

3
4 Similarly, potassium channel pores interact with peptide scorpion toxins with high
5 affinity, but most of the binding energy depends on two amino acid positions on the toxin
6 molecule though fifteen residues are likely buried upon binding (Goldstein, S.A. et al.,
7 *Neuron* 12: 1377-1388, 1994; Hidalgo, P. and MacKinnon, R., *Science* 268: 307-310,
8 1995; Ranganathan, R. et al., *Neuron* 16: 131-139, 1996; Stampe, P. et al., *Biochemistry*
9 33: 443-450, 1994). Thus, protein interaction surfaces contain functional epitopes or "hot
10 spots" of binding energy that are generally not predictable from the atomic structure.

11
12 In addition, a large body of evidence suggests that the change in free energy at a
13 protein interaction surface propagates through the tertiary structure in a seemingly
14 arbitrary manner. For example, studies addressing mechanisms of substrate specificity in
15 serine proteases show that many positions distantly positioned from the active site
16 contribute to determining the energetics of catalytic residues (Hedstrom, L., *Biol. Chem.*
17 377: 465-470, 1996; Hedstrom, L. et al., *Science* 255: 1249-1253, 1992; Perona, J.J. et
18 al., *Biochemistry* 34: 1489-1499, 1995).

19
20 Indeed, the conversion of trypsin to chymotrypsin specificity required a large set
21 of simultaneous mutations, many at unexpected positions. Similarly, mutations
22 introduced during maturation of antibody specificity have been shown to occur at sites
23 distant in tertiary structure from the antigen-binding site despite substantial increases in
24 binding energy (Patten, P.A. et al., *Science* 271: 1086-1091, 1996). Thus, protein
25 function appears to depend on the energetic interactions of a set of amino acid positions
26 that are structurally dispersed and that, like binding hot spots, are unpredictable from
27 even high-resolution crystal structures.

28
29 One potential approach to mapping these energetic interactions in a protein is
30 through massive mutagenesis. Indeed, thermodynamic mutant cycle analysis (Hidalgo, P.
31 and MacKinnon, R., *Science* 268: 307-310, 1995; Carter, P.J. et al., *Cell* 38: 835-840,

1 1984; Schreiber, G. and Fersht, A.R., *J. Mol. Biol.* 248: 478-486, 1995), a technique that
2 measures the energetic interaction of two mutations, provides a direct method to
3 systematically probe energetic relationships of protein sites. However, practical
4 considerations, such as the number of mutants that can be reasonably generated and
5 studied per unit time in the laboratory, limit this technique to small-scale studies,
6 obviating a full mapping of all energetic interactions on a complete protein.

7

8 Statistical methods have been reported for the analysis of biological sequences,
9 typically in the determination of homologous protein families and evolutionary
10 conservation.

11

12 Ortiz, A.R. et al. (*Pac. Symp. Biocomput.*, 316-327, 1997) describes a method of
13 predicting the low resolution three dimensional structure of proteins starting from a
14 multiple sequence alignment. Secondary structure predictions and minimized Monte
15 Carlo energy calculations are used to predict protein structures.

16

17 Sunyaev, S.R. et al. (*Protein Eng.*, 12: 387-394, 1999) describes the use of
18 position-specific independent counts at a given position in a sequence alignment in
19 identifying distantly related protein sequences.

20

21 Karlin, S. and Brendel, V. (*Science*, 257: 39-49, 1992) discuss the use of
22 statistical methods for characterizing anomalies in sequences, for determining
23 compositional biases in proteins, and for analyzing spacings of sequence markers. Karlin
24 (*Curr. Opin. Struct. Biol.*, 5: 360-371, 1995; *Philos. Trans. R. Soc. Lond. B. Biol. Sci.*
25 344: 391-402, 1994) further describes the use of statistical methods for the identification
26 of common segments between protein sequences, and the use of distributional theory in
27 multiple sequence alignments.

28

29 Bailey, T.L. and Gribskov, M. (*Bioinformatics*, 14: 48-54, 1998) propose the use
30 of the QFAST statistical algorithm for accurate and sensitive sequence homology
31 searches.

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1

2 Hughey, R. and Krogh, A. (*Comput. Appl. Biosci.* 12: 95-107, 1996) discuss the
3 use of Hidden Markov models (HMMs) to identify protein sequences with a given
4 domain, or to perform a multiple alignment of sequences.

5

6 Vingron, M. and Waterman, M.S. (*J. Mol. Biol.* 235: 1-12, 1994) describe
7 statistical analyses of DNA and protein alignments. Statistics are used to optimize
8 alignment parameters.

9

10 Leluk, J. (*Comput. Chem.* 22(1):123-131, 1998) describes statistical analyses of
11 proteins taking advantage of the correlation between amino acids and their corresponding
12 DNA codons. The analyses are useful for determining corresponding sequences between
13 proteins, and for investigating evolutionary divergence between proteins.

14

15 Bohm, G. and Jaenicke, R. (*Protein Sci.* 1: 1269-1278, 1992) propose the use of
16 statistical methods for the discrimination between native protein three dimensional
17 structures and corresponding misfolded structures.

18

19 U.S. Patent No. 5,523,208 (issued June 4, 1996) discusses the use of amino acid
20 hydropathy values to search protein databases for proteins predicted to interact with each
21 other.

22

23 The foregoing shows that a need exists for improved methods for the
24 identification of evolutionarily-conserved and interacting positions in biological
25 sequences, such as interacting amino acid positions in protein sequences. The
26 identification of evolutionarily-conserved amino acid positions may be used to identify
27 key regions in the protein for protein-drug interactions, to identify potential sites in
28 proteins that lead to hereditary mutation diseases, and the identification of catalytic sites
29 to improve enzyme activities, to name but several examples. The identification of
30 interacting amino acid positions is useful to predict how a protein folds into a three
31 dimensional structure, to predict how distant sites may interact to form a catalytic active

1 site in an enzyme, and to predict effects of a drug interaction with an amino acid position
2 may affect other amino acid positions, to name but a few examples.

3

4 **SUMMARY OF THE INVENTION**

5

6 The invention relates to a statistical method for the analysis of biological
7 sequences. The invention is useful to identify a) positions in biological sequences that
8 appear to be evolutionarily conserved, and b) positions in biological sequences that
9 appear to interact with one another. In addition, the invention is useful to identify c) the
10 functions of the pathways between interacting positions, and d) the mechanisms
11 responsible for those pathways, or connections. The invention may be used for any
12 biological sequence, including proteins, ribonucleic acids (RNA), deoxyribonucleic acids
13 (DNA), lipids, and polysaccharides (sugars), to name but a few examples. The invention
14 is believed to be particularly useful in the analysis of protein sequences.

15

16 The present methods are preferably performed by a suitably programmed
17 machine. For illustration, the following description and examples involve the use of
18 protein/amino acid sequences, but those skilled in the art having the benefit of this
19 disclosure will recognize that the same approach may be used for other biological
20 sequences, as described in greater detail near the end of this disclosure.

21

22 A set of amino acid sequences that are members of a common structural family is
23 provided; those amino acid sequences are aligned to produce a multiple sequence
24 alignment (MSA). For each position i in the multiple sequence alignment, a conservation
25 energy value (ΔG^{stat}) is calculated.

26

27 The respective conservation energy values represent the overall deviation of
28 amino acid frequencies, at the respective positions, from the mean values (i.e., the
29 expected values) for the amino acids in question. A list of positions with statistically
30 significant conservation energy values is generated. The conservation energy values may

1 be displayed in a graphical image (e.g., a bar graph or a three dimensional map) to aid
2 analysis.

3

4 To determine interacting positions, a specific position within the multiple
5 sequence alignment that has a statistically significant conservation energy value is
6 selected. A subset of the full set of amino acid sequences is selected. The subset is
7 analyzed and the vector difference between ΔG^{stat} of the subset and the ΔG^{stat} obtained
8 from the larger full set of sequences is calculated. This vector difference ($\Delta \Delta G_{i,j}^{\text{stat}}$)
9 represents the degree to which the probability of individual amino acids at position i is
10 dependent on the perturbation at position j. This difference value may be displayed in a
11 graphical image (e.g. a bar graph or a three dimensional map) to aid analysis.

12

13 In one respect, the invention is a method of identifying one or more positions in a
14 polymer family. The method includes accessing data representing a multiple sequence
15 alignment (MSA) of a plurality of polymer sequences. The method also includes
16 identifying one or more positions within the MSA that have statistically significant
17 conservation energy values using the following equation:

18

$$\Delta G_i^{\text{stat}} = kT^* \sqrt{\sum_x \left(\ln \frac{P_i^x}{P_{MSA}^x} \right)^2}$$

19

wherein:

20 i is a position in the MSA;

21 ΔG_i^{stat} is the conservation energy value for position i;

22 P_i^x is the probability of monomer x at position i;

23 P_{MSA}^x is the probability of monomer x in the MSA; and

24 kT^* is an energy unit, where k is Boltzmann's constant.

25

26 In other aspects, the method may be executed using a machine. The invention
27 may be a program storage device readable by the machine and encoding instructions
28 executable by the machine for performing the steps described above. The method may
29 include generating a graphical image of the conservation energy values (which is

1 described below in greater detail). The polymer sequences may be protein sequences.
2 Monomer x may be amino acid x. The data accessed may be data from the PDZ domain
3 family. The data accessed may also be data from the p21^{ras} domain family. The data
4 accessed may also be from the hemoglobin domain family.

5

6 In another respect, the invention is a method of identifying one or more positions
7 in a polymer family. The method includes accessing data representing a multiple
8 sequence alignment (MSA) of a plurality of polymer sequences. The method also
9 includes calculating a conservation energy value for each position in the MSA using the
10 following equation:

$$11 \quad \Delta G_i^{stat} = kT^* \sqrt{\sum_x \left(\ln \frac{P_i^x}{P_{MSA}^x} \right)^2}$$

12 wherein:

13 i is a position in the MSA;

14 ΔG_i^{stat} is the conservation energy value for position i;

15 P_i^x is the probability of monomer x at position i;

16 P_{MSA}^x is the probability of monomer x in the MSA; and

17 kT* is an energy unit, where k is Boltzmann's constant.

18 The method also includes identifying one or more positions within the MSA that have
19 statistically significant conservation energy values.

20

21 In other aspects, the method may be executed using a machine. The invention
22 may be a program storage device readable by the machine and encoding instructions
23 executable by the machine for performing the steps described above. The method may
24 include generating a graphical image of the conservation energy values (which is
25 described below in greater detail). The polymer sequences may be protein sequences.
26 Monomer x may be amino acid x. The data accessed may be data from the PDZ domain
27 family. The data accessed may also be data from the p21^{ras} domain family. The data
28 accessed may also be from the hemoglobin domain family.

29

1 In another respect, the invention is a machine-executed method of quantitatively
2 identifying one or more amino acid positions in a protein family that are suspected to be
3 evolutionarily conserved. The method includes accessing data representing a multiple
4 sequence alignment (MSA) of a plurality of protein sequences that are members of a
5 common structural family. The method also includes for each position in the MSA,
6 calculating a respective conservation energy value using the following equation:

$$7 \quad \Delta G_i^{stat} = kT^* \sqrt{\sum_x \left(\ln \frac{P_i^x}{P_{MSA}^x} \right)^2}$$

8 wherein:

9 i is a position in the MSA;

10 ΔG_i^{stat} is the conservation energy value for position i;

11 P_i^x is the probability of amino acid x at position i;

12 P_{MSA}^x is the probability of amino acid x in the MSA; and

13 kT^* is an energy unit, where k is Boltzmann's constant; and

14 The method also includes identifying one or more positions within the MSA that have
15 statistically significant conservation energy values.

16

17 In another respect, the invention is a method useful in identifying interacting
18 monomers in a polymer family. The method includes accessing data representing a
19 multiple sequence alignment (MSA) of a plurality of polymer sequences. The method
20 also includes calculating a respective conservation energy value for each position in the
21 MSA using the following equation:

$$22 \quad \Delta G_i^{stat} = kT^* \sqrt{\sum_x \left(\ln \frac{P_i^x}{P_{MSA}^x} \right)^2}$$

23

 wherein:

24 i is a position in the MSA;

25 ΔG_i^{stat} is the conservation energy value for position i;

26 P_i^x is the probability of monomer x at position i;

1 P_{MSA}^x is the probability of monomer x in the MSA; and
2 kT* is an energy unit, where k is Boltzmann's constant.
3 The method includes perturbing a position in the MSA other than position i; re-
4 calculating the respective conservation energy value for each position in the MSA to
5 yield a perturbed conservation energy value; and identifying positions within the MSA
6 that have statistically significant differences between their respective conservation energy
7 values and their perturbed conservation energy values.

8
9 In other aspects, the perturbing may include selecting a position j in the MSA; and
10 selecting a subset of the MSA, the subset having one or more monomers at position j in
11 the MSA. The re-calculating and identifying may include for each position in the MSA,
12 calculating a vector difference $\Delta\Delta G^{stat}$ between the conservation energy value of the
13 MSA and a conservation energy value of the subset of the MSA using the following
14 equation:

15

$$\Delta\Delta G_{i,j}^{stat} = kT^* \sqrt{\sum_x \left(\ln \frac{P_{i|\tilde{S}}^x}{P_{MSA|\tilde{S}}^x} - \ln \frac{P_i^x}{P_{MSA}^x} \right)^2}$$

16 wherein:

17 $\Delta\Delta G_{i,j}^{stat}$ is the vector difference in conservation energy values for
18 position i;

19 $P_{i|\tilde{S}}^x$ is the probability of monomer x at position i of the subset; and

20 $P_{MSA|\tilde{S}}^x$ is the probability of monomer x in the subset.

21 The method may also include identifying positions within the MSA that have statistically
22 significant $\Delta\Delta G^{stat}$ values.

23
24 In still other aspects, the method may include generating a graphical image of the
25 $\Delta\Delta G^{stat}$ values. The method may be executed using a machine. The invention may be a
26 program storage device readable by the machine and encoding instructions executable by
27 the machine for performing the steps of accessing, calculating, perturbing, re-calculating,

1 and identify recited above. The polymer sequences may be protein sequences. Monomer
2 x may be amino acid x. The data accessed may be data from the PDZ domain family.
3 The data accessed may be data from the p21^{ras} domain family. The data accessed may be
4 data from the hemoglobin domain family.

5

6 In another respect, the invention is a machine-executed method of quantitatively
7 identifying interacting amino acids in a protein family. The method includes accessing
8 data representing a multiple sequence alignment (MSA) of a plurality of protein
9 sequences that are members of a common structural family. The method also includes
10 for each position in the MSA, calculating a respective conservation energy value using
11 the following equation:

$$12 \quad \Delta G_i^{stat} = kT^* \sqrt{\sum_x \left(\ln \frac{P_i^x}{P_{MSA}^x} \right)^2}$$

13 wherein:

14 i is a position in the MSA;

15 ΔG_i^{stat} is the conservation energy value for position i;

16 P_i^x is the probability of amino acid x at position i;

17 P_{MSA}^x is the probability of amino acid x in the MSA; and

18 kT^* is an energy unit, where k is Boltzmann's constant.

19 The method includes selecting a position j in the MSA; selecting a subset of the MSA,
20 wherein the subset has one or more amino acids at position j in the multiple sequence
21 alignment; for each position in the multiple sequence alignment, calculating a vector
22 difference between the respective conservation energy value of the multiple sequence
23 alignment and the respective conservation energy value of the subset of the multiple
24 sequence alignment; and identifying positions within the MSA that have statistically
25 significant vector differences.

26

27 In another respect, the invention is a method of analyzing data that includes
28 providing at least one protein having a crystal structure and multiple positions; solving

1 the crystal structure of the at least one protein; and identifying pathways between
2 interacting positions on the at least one protein.

3

4 In another respect, the invention is a method of analyzing the effect of
5 perturbation on a protein that includes accessing data representing at least one protein and
6 at least one perturbed protein. Both proteins have at least one atom that is identical, or
7 the same. The method also includes calculating a quantity of change Δ_{struct} to the atom
8 using the following equation:

9

$$\Delta_{struct} = \frac{|\vec{r}_{mut}|}{\sqrt{\sigma_{mut}^2 + \sigma_{wt}^2}}$$

10 wherein:

11 $|\vec{r}_{mut}|$ is the magnitude of a vector connecting the position of the
12 atom in the at least one perturbed protein and the position
13 of the atom in the at least one protein;

14 σ_{mut} is a standard deviation of the atom in the at least one
15 perturbed protein; and

16 σ_{wt} is a standard deviation of the atom in the at least one protein.

17

18 In another respect, the invention is a method of analyzing data that includes
19 accessing data representing at least one protein, a first perturbation of the at least one
20 protein yielding a first perturbed protein, a second perturbation of the at least one protein
21 yielding a second perturbed protein, and a double perturbation of the at least one protein
22 yielding a double perturbed protein, the double perturbation comprising both the first and
23 second perturbations. The proteins each have at least one identical atom. The method
24 also includes calculating a quantity of structural coupling $\Delta\Delta_{struct}$ between the first and
25 second perturbations using the following equation:

26

$$\Delta\Delta_{struct} = \frac{|\vec{r}_{mut1} - \vec{r}_{mut1|mut2}|}{\sqrt{\sigma_{wt}^2 + \sigma_{mut1}^2 + \sigma_{mut2}^2 + \sigma_{mut1,mut2}^2}}$$

CROSS-REFERENCE TO RELATED APPLICATIONS

1 wherein:

2 \vec{r}_{mut_1} is a vector connecting the position of the atom in the first
3 perturbed protein and the position of the atom in the at least
4 one protein;

5 \vec{r}_{mut_1,mut_2} is a vector connecting the position of the atom in the
6 double perturbed protein and the position of the atom in the
7 second perturbed protein;

8 σ_{wt} is a standard deviation of the atom in the at least one protein;

9 σ_{mut_1} is a standard deviation of the atom in the first perturbed
10 protein;

11 σ_{mut_2} is a standard deviation of the atom in the second perturbed
12 protein; and

13 σ_{mut_1,mut_2} is a standard deviation of the atom in the double
14 perturbed protein.

15

16 In another respect, the invention is a method of analyzing microarray data that
17 includes accessing microarray data representing an expression level of at least one gene,
18 an expression level of the at least one gene resulting from a first perturbation, an
19 expression level of the at least one gene resulting from a second perturbation, and an
20 expression level of the at least one gene resulting from a double perturbation comprising
21 both the first and second perturbations. The method also includes calculating a degree of
22 coupling $\Delta\Delta E$ between the first and second perturbations using the following equation:

23

$$\Delta\Delta E = kT' \ln\left(\frac{f_1}{f_2}\right)$$

24

wherein:

25 f_1 is the fold effect of the gene due to the first perturbation relative
26 to the at least one gene;

27 f_2 is the fold effect of the gene due to the double perturbation
28 relative to the second perturbation; and

kT' is an energy unit, where k is Boltzmann's constant.

BRIEF DESCRIPTION OF THE DRAWINGS

The following figures form part of the present specification and are included to further demonstrate certain aspects of the present invention. The invention may be better understood by reference to one or more of these drawings in combination with the detailed description of specific embodiments presented herein.

Figure	Description
1	Histograms of amino acids for all 36,498 entries in the Swiss-Prot database (as of 10/98) and for 274 members of the PDZ protein family. Black bars represent all Swiss-Prot proteins, gray bars represent the PDZ protein family.
2	Histogram of amino acids at position 76 of PDZ multiple sequence alignment. Black bars represent all Swiss-Prot proteins, gray bars represent position 76. Position 76 is highly conserved, as evidenced by the high distribution values (Y-axis).
3	Histogram of amino acids at position 99 of PDZ multiple sequence alignment. Black bars represent all Swiss-Prot proteins, gray bars represent position 99. Position 99 is weakly conserved, as evidenced by the low distribution values (Y-axis).
4	Calculated ΔG^{stat} for all positions in PDZ multiple sequence alignment. The statistical energy (ΔG^{stat}) representing evolutionary conservation is plotted against the primary structure position.
5	Thermodynamic cycle describing statistical coupling.
6	Thermodynamic cycle describing mutational coupling.
7	Amino acid distributions at positions 67 and 34 before (black bars) and after (gray bars) a 6.45 kT^* perturbation at position 76. Note that the distribution at position 67 changes very little upon perturbation at position 76 despite high overall conservation, and that the distribution at position 34 changes significantly.
8	A full mapping of $\Delta\Delta G_{i,j}^{\text{stat}}$ for PDZ position 76 for all other positions in the fold family. Only a small set of coupled positions distributed throughout the primary sequence emerge above noise.
9	Statistical coupling ($\Delta\Delta G^{\text{stat}}$) with sites categorized in three groups: sites that are statistically coupled and near to position 76 [33,34,39,80,84], sites that are statistically coupled but distant from position 76 [26,29,66,67,90], and sites that are statistically uncoupled [32,44,75,89].
10	Mutational coupling ($\Delta\Delta G^{\text{mut}}$), with sites categorized in three groups: sites that are statistically coupled and near to position 76 [33,34,39,80,84], sites that are

Figure	Description
	statistically coupled but distant from position 76 [26,29,66,67,90], and sites that are statistically uncoupled [32,44,75,89]. Inset is a binding isotherm for wild-type PDZ3 ^{psd95} protein and a class I binding peptide. An average and standard deviation of five measurements are shown for each ligand concentration tested, with the smooth curve showing a fit to the Hill equation.
11	Scatter plot of mutational coupling energies and statistical coupling energies. This plot demonstrates good prediction of thermodynamic coupling through the statistical analysis.
12	Thermodynamic mutant cycle analysis between mutations at PDZ position 76 (H76Y) and mutations at ligand positions at the directly-interacting position (T7F) and at the carboxyl-terminal position (V9A). This suggests coupling of both peptide positions with PDZ position 76.

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DEFINITIONS

The following definitions are provided in order to aid those skilled in the art in understanding the detailed description of the present invention.

“Evolutionarily conserved amino acid positions” refers to particular positions within a multiple sequence alignment which display a non-zero ΔG^{stat} as calculated by Equation 2. In general terms, this refers to positions within a sequence that have a non-random distribution of monomers. For example, if many members of a protein family have histidine at position 50, this would suggest that having histidine at position 50 is important for the protein’s function, and that it has been conserved during evolution. Conversely, if position 50 in the members of the protein family displayed a random distribution of amino acids, this would suggest that there was no requirement for any particular amino acids at this position during evolution.

“Multiple sequence alignment” (MSA) refers to an optimized alignment of two or more sequences. Protein multiple sequence alignments may be performed manually or by computer programs, e.g. CLUSTALW (Thompson, et al. *Nucl. Acids Res.*, 22: 4673-4680, 1994). Multiple sequence alignments performed by computer programs may be subsequently modified manually if more detailed structural information is known about the protein sequence or structure.

1

2 “Protein sequence” and “amino acid sequence” refer to the amino acid sequence
3 that constitutes a protein. Amino acids are commonly referred to by their one letter
4 abbreviations: Alanine, A; Cysteine, C; Aspartic acid, D; Glutamic acid, E;
5 Phenylalanine, F; Glycine, G; Histidine, H; Isoleucine, I; Lysine, K; Leucine, L;
6 Methionine, M; Asparagine, N; Proline, P; Glutamine, Q; Arginine, R; Serine, S;
7 Threonine, T; Valine, V; Tryptophan, W; Tyrosine, Y.

8

9 “Protein family” or “structural family” refers to a set of protein sequences that
10 may be aligned. The protein family may have the same biological or enzymatic function,
11 (e.g., a set of DNA polymerases or glutamate dehydrogenases), or a common structural
12 region (e.g., a set of proteins containing a zinc finger region).

13

14 “Statistically significant conservation energy values” may vary with the
15 application. In general, this refers to values that are greater than the background “noise”
16 value. One manner of arriving at values that are greater than the background noise is to
17 fit the set of energy values for all positions in an alignment to well-established Gaussian
18 error models. Values greater than two standard deviations from the mean may be
19 classified as “statistically significant.”

20

21 **DESCRIPTION OF ILLUSTRATIVE EMBODIMENTS**

22

23 The illustrative method described below incorporates the essential features of the
24 evolutionary process in two guiding principles: (1) positions on the protein that are not
25 constrained by the energetic requirements of folding or function should show an amino-
26 acid distribution that approaches the mean overall amino-acid distributions found in all
27 natural proteins; and (2) the conserved functional interaction of two positions in any
28 protein family should make the amino-acid distributions dependent on each other; thus,
29 the outcome at one position should influence the outcome at the other coupled position. A
30 corollary of (1) is that conservation at any given position can be quantitatively regarded
31 as the degree to which the amino-acid distribution at that position deviates from the mean

1 distribution in all proteins. The folding of a protein is the process by which the linear
2 amino-acid sequence of a protein generates the three-dimensional structure of the protein.

3

4 A first step is the calculation of conservation at each position in a multiple
5 sequence alignment. Each position on the sequence alignment may be characterized by a
6 vector of amino-acid frequencies:

7 $f_i = (f_{ala}, f_{cys}, \dots, f_{tyr})$

8 (Equation 1)

9 In the limit where an infinity of observed sequences is available for analysis, this
10 vector should just be the probabilities of each amino acid at position i . Since one
11 normally has only several hundred sequences of each protein family at best, the
12 probabilities given these observed frequencies are estimated using probability theory.
13 The binomial distribution gives the probability of n observations of amino acid x out of a
14 total of N sequences when the mean probability of amino acid x is p_x :

15
$$P_i^x = \frac{N!}{n_x!(N-n_x)!} p_x^{n_x} (1-p_x)^{N-n_x}$$

16 (Equation 2)

17 Thus the frequency vector may be converted to a probability vector for site i by
18 using this equation for each element of the vector of amino acid frequencies.

19

20 In order to investigate the energetic interactions of sites on a protein, it is
21 preferable for the statistical parameters to also have energy-like characteristics. This
22 greatly simplifies the interpretation of the data, especially in drawing the conceptual
23 analogy of this method to mutagenesis in proteins. The Boltzmann distribution of
24 classical thermodynamics gives the relationship of the relative probability of two states (i
25 and j) of a system to the statistical energy ($\Delta G_{i \rightarrow j}^x$) separating these states:

1
$$\frac{P_i^x}{P_j^x} = e^{-\frac{\Delta G_{i-j}^x}{kT^*}}$$

2 (Equation 3)

3 Using this equation, the probability vector is converted to a vector of statistical
4 energies where each element is now the statistical energy representing the deviance of
5 each amino acid from the mean value expected for all proteins. The magnitude of this
6 vector is the empirical parameter (in energetic units, kT^*) that quantitatively represents
7 conservation at any given site i of a sequence alignment:

8
$$\Delta G_i^{stat} = kT^* \sqrt{\sum_x \left(\ln \frac{P_i^x}{P_{MSA}^x} \right)^2}$$

9 (Equation 4)

10 This analysis may be used, for example, to identify the active site (the functional
11 surface), binding site, or allosteric site of a protein.

12 An additional embodiment of the invention is the subsequent energetic
13 measurement of coupling of two positions on a protein. This amounts to determining
14 whether the amino acid frequencies at one site are affected by changes at another site. To
15 address this, a change is made to the observed amino acid frequencies at one site j by
16 selecting out a subset of the sequence alignment. This selecting out causes a change in
17 the frequencies at site j . For example, if a position started with 0.6H and 0.4V, selecting
18 out all sequences that have only H at that site would have the effect of changing the
19 frequencies at that site to 1.0H. After making such a selection, the vector of statistical
20 energies is then re-calculated at each position i of the subset alignment. The difference in
21 the statistical energy vector at a site i before and after the change at j is a measure of the
22 interdependency of the two sites. This is intuitive in that if site i were totally independent
23 of j , then any change made at j is very unlikely to result in any change at i . The coupling
24 between sites i and j is calculated as the magnitude of the difference vector at i before and
25 after the perturbation at site j .

$$\Delta\Delta G_{i,j}^{stat} = kT^* \sqrt{\sum_x \left(\ln \frac{P_{i|\delta}^x}{P_{MSA|\delta}^x} - \ln \frac{P_i^x}{P_{MSA}^x} \right)^2}$$

2 (Equation 5)

3 MEAN DISTRIBUTION OF AMINO ACIDS

4 In nature, the twenty naturally occurring amino acids are not used equally. The
5 mean distributions of amino acids may be obtained from scientific publications, the
6 internet, or may be generated from a suitable database such as PIR, GenBank, or
7 SwissPROT. In order to generate mean distributions, a collection of proteins is selected,
8 and the occurrence of each amino acid is calculated as a decimal fraction of the total
9 number of amino acid residues in the collection. For example, if a selected collection of
10 300 protein sequences containing a total of 300,000 amino acid residues has 21,477
11 glycines, the mean frequency of glycine would be calculated to be 0.07159
12 (21477/300000).

13 MULTIPLE SEQUENCE ALIGNMENTS

14 Protein sequences may be aligned to optimize the alignment of identical or similar
15 amino acids, affording a “multiple sequence alignment” representing similar three
16 dimensional structures. Multiple sequence alignments may be performed manually, or
17 preferably by a computer program such as CLUSTALW or other commercial or publicly-
18 available programs.

19 STATISTICAL ANALYSIS OF CONSERVATION

20 For an evolutionarily well-sampled multiple sequence alignment, where adding
21 additional sequences does not change the distribution at sites much, the probability of any
22 amino acid x at site i relative to the probability of the amino acid at another site, j , is
23 related to the statistical free energy separating i and j for the x^{th} amino acid ($\Delta G_{i \rightarrow j}^x$) by
24 the Boltzmann distribution computed in accordance with Equation 3 (Tolman, R.C. *The*
25 *Principles of Statistical Mechanics* (Dover Publications Inc., New York, 1938), where

1 kT^* is an arbitrary energy unit. For conventional statistical mechanical systems at
2 equilibrium, the temperature (T) of an ensemble is proportional to the mean velocity of
3 state transitions, and defines the fundamental energy unit kT , where k is Boltzmann's
4 constant. In our analysis, we treat sites on a multiple sequence alignment as individual
5 statistical mechanical systems that can be represented as discrete states in an overall state
6 space of amino-acid frequencies. The "temperature" (T^*) of an ensemble of such
7 systems is again related to the mean transition rates between states, but we note that the
8 energy unit in such a system (kT^*) is not necessarily related to that for conventional
9 mechanical systems.

10

11 The probability of any amino acid x at site i (P_i^x) is given by the binomial
12 probability of the observed number of x^{th} amino acids given its mean frequency in all
13 proteins. The full distribution of amino acids at a site can then be characterized by a
14 twenty-element vector of P_i^x for all x ($\overrightarrow{P_i^x}$). Looking at a hypothetical site where all
15 amino acids are found at their mean frequencies in the MSA as a reference state for all
16 sites, Equation 3 may be used to transform $\overrightarrow{P_i^x}$ into a vector of statistical energies which
17 represents the evolutionary constraint at site i . An overall empirical evolutionary
18 conservation parameter (ΔG^{stat}) is defined for site i per Equation 4.

19

20 For each position in the generated multiple sequence alignment, ΔG^{stat} is
21 calculated using Equation 4. A list of positions within the multiple sequence alignment
22 having statistically significant conservation energy values is generated. That is, one may
23 identify the position or positions within the MSA that have statistically significant
24 conservation energy values. As explained above, this may be achieved by fitting the set
25 of energy values for all positions in the MSA to well-established Gaussian error models.
26 Values greater than two standard deviations from the mean may be classified as
27 statistically significant. Optionally, a graphical display of the conservation energy values
28 may be generated using commercial or publicly available graphing software.

1 **STATISTICAL ANALYSIS OF COUPLING**

2 Functional coupling of sites should mutually constrain the evolution of those sites,
3 and therefore their amino acid distributions in a sequence alignment should be
4 statistically correlated. To measure this, the conservation energy value at a given site i is
5 measured under two conditions: (1) the full set multiple sequence alignment, and (2) a
6 selected subset of the multiple sequence alignment representing a perturbation of the
7 amino acid frequencies at another site j. The magnitude of the difference in these two
8 energy values ($\Delta\Delta G_{i,j}^{\text{stat}}$) quantitatively represents the degree to which the probability of
9 individual amino acids at position i is dependent on the perturbation at position j (see
10 Equation 5).

11
12 The multinomial probability for all 20 amino acids provides the overall
13 probability of observing a given amino acid distribution at a site, but is degenerate given
14 redistribution of amino acids with similar mean frequency. This suggests that even
15 significant changes in the amino acid composition at one site upon perturbation at another
16 may go unrecognized if measured as changes in this value. For example, consider a site
17 that displays a distribution 0.4 Ala, 0.4 Asp, 0.2 Ile in the overall alignment, and which
18 changes to 0.4 Ala, 0.2 Asp, 0.4 Ile upon perturbation at another site. Since the mean
19 frequency of Asp and Ile is nearly identical (Figure 1), the multinomial probability of
20 these two distributions are the same though the significant reorganization of chemical
21 character suggests that these positions are indeed coupled. The inventive method
22 described accounts for all such cases by treating sites as vectors of individual amino acid
23 probabilities, where each amino acid distribution maps to a unique vector.

24 The following Examples are included to demonstrate preferred embodiments of
25 the invention. It should be appreciated by those of skill in the art that the techniques
26 disclosed in the Examples which follow represent techniques discovered by the inventors
27 to function well in the practice of the invention, and thus can be considered to constitute
28 preferred modes for its practice. However, those of skill in the art should, in light of the
29 present disclosure, appreciate that many changes can be made in the specific

1 embodiments which are disclosed and still obtain a like or similar result without
2 departing from the spirit and scope of the invention.

3 **EXAMPLES**

4 **EXAMPLE 1: COMPUTATIONS USING SOFTWARE**

5 Software implementing the approach described above was written in C for DEC
6 alpha platforms running DEC Unix. A copy of the source code is reproduced in the
7 Appendix. For calculation of the mean frequencies of amino acids, we selected all
8 eukaryotic sequences from the Swiss-Prot database (as of October 1998) parsed for
9 truncation of the pre/pro sequences, and made histograms (i.e., graphs) of amino-acid
10 frequencies. Statistical energies at positions in a multiple sequence alignment are
11 calculated as follows. Each position in a multiple sequence alignment can be described
12 as a twenty-element vector of individual amino acid frequencies. Each element is
13 transformed into a probability for that amino acid using the binomial density function:

14
$$P(x) = \frac{N!}{x!(N-x)!} p^x (1-p)^{N-x}$$

15 (Equation 6)

16 where N is the total number of sequences, x is the number of sequences with a given
17 amino acid, and p is the mean frequency of that amino acid in all proteins.

18
19 Each element in the probability vector is then converted to a statistical energy for
20 that amino acid using Equation 4, where a hypothetical site where the amino acids are
21 found at their mean frequency in the multiple sequence alignment is taken as the
22 reference state. The scalar statistical energy of conservation for a site (ΔG_i^{stat}) is given by
23 the magnitude of the statistical energy vector. Equation 4 summarizes the conversion of
24 amino acid probabilities to ΔG_i^{stat} . Stirling's approximation was used for evaluation of
25 large factorials (>170). For visualization and analysis, statistical energies were arbitrarily
26 scaled by 0.01 for compatibility with GRASP, and outputted in Excel (Microsoft) format
27 or written to a protein data bank (PDB) file of a representative member of the fold family.

1 A fold family is a group of proteins that share an overall three-dimensional structure.
2 Mapping of statistical energies onto tertiary structures was done using GRASP (Nicholls,
3 A. et al., *Proteins* 11: 281, 1999). As used herein, “tertiary structures” are essentially
4 synonymous with three-dimensional structures for single protein chains.

5 **EXAMPLE 2: LABORATORY METHODS**

6 Fluorescence energy transfer experiments were carried out using a luminescence
7 spectrometer (Perkin Elmer LS 50 B). A final concentration of 100 nM EGFP-PDZ
8 fusion protein in storage buffer was used for peptide titrations. EGFP was excited at
9 475 nm and emission was measured at 508 nm. Ligand peptides were synthesized with
10 an N-terminal tetramethylrhodamine adduct, and were freshly diluted from a single batch
11 of 6 μ M frozen aliquots for binding measurements. For all measurements, we used the
12 following binding peptide (or mutants thereof, as indicated) co-crystallized in the original
13 structure determination. Energy transfer was followed by quenching of fluorescence at
14 508 nm, corrected for peptide fluorescence. Transfer efficiencies measured for four or
15 five peptide concentrations covering a two log-order range around the K_d were used for
16 each binding energy calculation; each individual measurement was made 3 to 5 times.
17 Data were fit to the Hill equation (Origin, MicroCal Software, Northampton, MA).

18

19 Site-directed mutagenesis on the rat PSD-95 third PDZ domain (residues 294-
20 402) was carried out using standard PCR-based techniques (Sambrook, J. et al. Molecular
21 Cloning: A Laboratory Manual, 2nd Ed., Cold Spring Harbor, NY, 1989). Domains were
22 expressed as C-terminal fusions with the enhanced green fluorescent protein (EGFP,
23 Heim, R. and Tsien, R.Y., *Curr. Biol.* 6: 178-181, 1996) using the pRSET-R vector
24 (Invitrogen, Carlsbad, CA) in *E. coli* (BL21(DE3), Stratagene, La Jolla, CA). In each
25 case, 500 mL cultures in TB+100 μ g/ml ampicillin were grown to an OD₆₀₀ of 1.2 at
26 37°C, induced for 4 hours with 100 μ M IPTG and harvested. Cells were lysed with 20
27 mL B-PER (Pierce) for 20 minutes at room temperature and centrifuged 20 minutes at
28 43,000 x g at 4°C. Protein was batch-bound to 0.5 mL bed volume of Ni-NTA agarose
29 beads (Qiagen, Valencia, CA) prewashed in binding buffer (25 mM Tris (pH 8.0), 500
30 mM NaCl, 10 mM imidazole) with 0.1% Tween-20, washed with 50 column volumes of

1 binding buffer, and eluted with Elution Buffer (50 mM Tris (pH 8.0), 1 M NaCl, 200 mM
2 imidazole). The protein was dialyzed overnight into storage buffer (50 mM Tris (pH
3 8.0), 100 mM NaCl, 1 mM DTT) at 4°C and used immediately for binding assays or flash
4 frozen and stored at -80°C for later use.

5 **EXAMPLE 3: CONSERVATION OF AMINO ACIDS IN PDZ DOMAINS**

6 To determine mean amino-acid frequencies in all proteins, histograms of amino
7 acids for all 36,498 entries in the Swiss-Prot database (as of October 1998) of eukaryotic
8 non-redundant proteins were created, and the mean values were calculated (Figure 1,
9 black bars). Since all structural and functional information has been scrambled in this
10 analysis, the frequencies of amino acids should represent that which is expected without
11 any functional evolutionary constraint.

12
13 The PDZ domain family was selected as one model system for the analyses
14 described below. PDZ domains are a family of small, evolutionarily well-represented
15 protein binding motifs for which four high-resolution structures of distantly related
16 members exist (Doyle, D.A. et al., *Cell* 85: 1067-1076, 1996; Cabral, J.H. et al., *Nature*
17 382: 649-652, 1996; Daniels, D.L. et al., *Nat. Struct. Biol.* 5: 317-325, 1998;
18 Ranganathan, R., unpublished results). The structures are remarkably similar (RMS
19 deviation in C_α atoms of 1.4 Å) though the average sequence identity between pairs of
20 domains is only 24%, and in many cases is indistinguishable from random. Structure-
21 based alignment techniques were used to generate a multiple sequence alignment of 274
22 eukaryotic PDZ domains.

23
24 Eukaryotic PDZ domains were collected from the non-redundant database using
25 PSI-BLAST (Altschul, S.F. et al., *Nucl. Acids Res.* 25: 3389-3402, 1997); four PDZ
26 domains with known structures (Doyle, D.A. et al., *Cell* 85: 1067-1076, 1996; Cabral,
27 J.H. et al., *Nature* 382: 649-652, 1996; Daniels, D.L. et al., *Nat. Struct. Biol.* 5: 317-325,
28 1998; Ranganathan, R., unpublished results) were used in initial searches. All non-
29 redundant PDZ domain sequences with an e-score equal to or less than 0.001 were
30 included for alignment. An initial alignment was created using PILEUP (Genetics

1 Computer Group, Madison, WI). Blocks of sequences with relatively high internal
2 homology were subjected to structure-based manual alignment (reviewed in Doolittle, R.
3 *Meth. Enzymol.* 266, 1996), and then aligned with homologous blocks. This process was
4 iterated until all blocks were aligned.

5

6 Interestingly, overall amino acid distributions for all proteins (Figure 1, black
7 bars) and for PDZ domains alone (Figure 1, gray bars) differ only modestly, a fact that
8 derives from the large sequence divergence of this fold family. Distributions at sites that
9 represent moderately conserved (Figure 2, Pos 76, $\Delta G^{\text{stat}} = 3.83 \text{ kT}^*$, $\sigma = 0.4 \text{ kT}^*$) and
10 weakly-conserved (Figure 3, Pos 99, $\Delta G^{\text{stat}} = 0.1 \text{ kT}^*$) positions show that even moderate
11 conservation skews the mean amino acid distribution significantly, and lack of
12 conservation is indeed correlated with distributions closer to the mean.

13

14 Equation 4 was used to calculate ΔG^{stat} for all positions on the PDZ domain
15 alignments. These data plotted on the primary structure show a dispersed pattern that
16 describes the overall energetic profile of the fold family (Figure 4). Not surprisingly, the
17 same data plotted on a representative three-dimensional structure of a member of the
18 family shows that this pattern simplifies into a rough description of the protein interaction
19 surface of the fold (Lichtarge, O. et al., *J. Mol. Biol.* 257: 342-358, 1996). For example,
20 the groove on the surface of the PDZ domain that contains the co-crystallized peptide
21 ligand (Doyle, D.A. et al., *Cell* 85: 1067-1076, 1996; Cabral, J.H. et al., *Nature* 382: 649-
22 652, 1996) emerges as the most conserved portion of the protein family. This finding is
23 consistent with the intuitive expectation that a proper measure of conservation should be
24 able to map functionally important sites on a protein.

25 **EXAMPLE 4: COUPLING OF AMINO ACIDS IN PDZ DOMAINS**

26 To characterize this energetic coupling function, one functionally important site in
27 the PDZ domain family was selected as a test case for the perturbation analysis. The
28 PDZ domain family is divided into distinct classes based on target sequence specificity;
29 class I domains bind to peptide ligands of the form $-S/T-X-V/I-COO^-$ where X represents
30 any amino acid, and class II domains bind to sequences of the form $-F/Y-X-V/A-COO^-$

1 (Songyang, Z. et al., *Science* 275: 73-77, 1997; Ponting, C.P. et al., *Bioessays* 19: 469-
2 479, 1997). An important determinant of ligand specificity is domain position 76 (Doyle,
3 D.A. et al., *Cell* 85: 1067-1076, 1996; the numbering scheme for the PDZ domain used is
4 consistent with that reported for the structures used for mapping statistical energies),
5 which appears to select the identity of the antepenultimate peptide position. In class I
6 domains, a histidine at this position hydrogen bonds to the serine or threonine hydroxyl
7 of the characteristic recognition motif (Doyle, D.A. et al., *Cell* 85: 1067-1076, 1996).

8

9 For analysis of statistical coupling, we selected sequences from the multiple
10 sequence alignment representing an alteration to the distribution of amino acids at one
11 site, and recalculated statistical energy vectors at all sites. For example, at PDZ position
12 76, we extracted the subset of sequences containing histidine at that position as the
13 “perturbed” multiple sequence alignment. The statistical coupling energy for site i given
14 a perturbation at j (δ_j) is the magnitude of the difference in energy vectors before and
15 after the perturbation (see Equation 5). All distributions were normalized for
16 comparison.

17

18 To examine the full pattern of energetic connectivity for PDZ position 76, we
19 made a perturbation to the amino acid distribution at this site by extracting the subset of
20 the multiple sequence alignment that contains only histidine at this position. The
21 statistical energetic consequence of this perturbation is a 6.45 kT^* change at position 76
22 from the full multiple sequence alignment. Figure 3a shows examples of amino acid
23 distributions for two PDZ positions that illustrate statistical coupling to position 76.
24 Position 63 is highly conserved in all PDZ domains, showing a distribution that is
25 virtually exclusive for leucine, isoleucine, or valine (Figure 7, upper panel), but one that
26 is largely unaffected by the perturbation at position 76. Consequently, this position
27 displays a low coupling energy ($\Delta\Delta G_{63,76}^{\text{stat}} = 0.31 \text{ kT}^*, \sigma = 0.3 \text{ kT}^*$) with respect to
28 position 76. In contrast, the distribution at position 34 changes for several amino acids
29 upon perturbation at position 76 (Figure 7, lower panel), resulting in significant statistical
30 coupling ($\Delta\Delta G_{80,76}^{\text{stat}} = 1.32 \text{ kT}^*$).

31

Figure 8 shows a full primary sequence mapping of statistical coupling for PDZ position 76. Interestingly, these data show that most positions in the fold family are not coupled to the perturbed site; instead, only a small set of statistical couplings emerges from noise. Mapping the data on the PDZ domain tertiary structure shows that the coupled sites fall into three classes. A small set of residues [positions 80, 84, 33, 34] are in the immediate environment of position 76, a finding consistent with expected local propagation of energy from a site of perturbation. In addition, other interaction surface residues implicated in target sequence recognition [positions 29, 26] emerge as coupled. This result suggests energy propagation through bound substrate, and would be an expected consequence of cooperative interaction of binding site residues. Finally, we observe unexpected coupling at long range from sites in the core and on the opposite side of the PDZ domain [positions 51, 57, 66].

EXAMPLE 5: MUTANT CYCLE STUDIES

To address the relationship of statistical coupling and the physical energetic coupling of sites, we used the technique of thermodynamic mutant cycle analysis (Hidalgo, P. and MacKinnon, R., *Science* 268: 307-310, 1995; Carter, P.J. et al., *Cell* 38: 835-840, 1984) to measure mutational coupling energies for position 76 for one PDZ domain ($\text{PDZ3}^{\text{psd-95}}$) and compared these data to the statistical predictions. In the mutant cycle method, the energetic effect of one mutation, m_1 , is measured for two conditions: (1) the wild-type background (ΔG_{m_1}) (Figure 6) or (2) the background of a second mutation, m_2 ($\Delta G_{m_1|m_2}$) (Figure 6). This method is analogous to the method of thermodynamic mutant cycle analysis as shown in Figure 5. The difference in these two energies gives the coupling energy ($\Delta\Delta G_{m_1,m_2}$) between the two mutations. Note that if m_1 does not have the same effect in condition 1 and 2 ($\Delta G_{m_1|m_2} \neq \Delta G_{m_1}$), then $\Delta\Delta G_{m_1,m_2}$ is non-zero and indicates thermodynamic coupling of the two mutations.

To follow energetic coupling, an equilibrium binding energy assay was developed based on fluorescence resonance energy transfer between green fluorescent protein (GFP)-PDZ domain fusion proteins and tetramethylrhodamine (TMR)-labeled interacting peptides. The inset in Figure 10 shows a binding isotherm for interaction a wild-type

1 GFP- PDZ3^{psd-95} protein and a TMR-labeled class I peptide, showing that this assay is
2 capable of high-resolution mapping of binding energies.

3

4 Using this assay, we measured coupling energies for a mutation at position 76
5 (H76Y) against mutations at a set of 14 PDZ domain positions and two peptide positions.
6 The mutations chosen were designed to test a range of statistical couplings on the PDZ
7 domain, including a set of sites that are not significantly statistically coupled. Figures 9-
8 show that statistical coupling energies at sites, whether spatially near to, or distant
9 from position 76 are in fact well correlated to the thermodynamic coupling through
10 mutagenesis. Importantly, statistically uncoupled sites display mutational coupling
11 energies near to noise. Figure 11 shows a scatter plot of these data comparing coupling
12 measured from statistical theory and from mutagenesis, indicating excellent reliability in
13 the assignment of thermodynamic coupling. Thus, patterns of statistical energetic
14 coupling for a protein site are likely to describe the thermodynamic energetic
15 connectivity for that position.

16

17 The statistical analysis for perturbation at position 76 indicated that other binding
18 site positions [positions 29 and 26] are energetically coupled, and suggested the
19 possibility of propagated coupling through the substrate peptide (Figure 8). Indeed,
20 mutations at the peptide position directly interacting with PDZ position 76 (T7F), and at
21 the position carrying the terminal carboxylate (V9A) are also thermodynamically coupled
22 to the H76Y mutation.

23 **EXAMPLE 6: APPLICATIONS TO NON-PROTEIN BIOLOGICAL
24 SEQUENCES**

25 The inventive methods may be used to analyze biological sequences other than
26 proteins. For example, ΔG_{stat} and $\Delta \Delta G_{i,j}^{\text{stat}}$ may be calculated for polysaccharides,
27 lipids, deoxyribonucleic acid (DNA, represented by A, C, G, and T bases), and
28 ribonucleic acid sequences (RNA, represented by A, C, G, and U bases) to identify
29 evolutionary conservation and interacting pairs of components. Any polymer of
30 monomers may be analyzed with the inventive methods. Application of the inventive

1 methods is not limited to biological sequences, as it may be applied to chemical
2 polymers, drugs, and other compounds.

3

4 The inventive methods may also be used to analyze inter-protein (two proteins)
5 interactions, as well as the intra-protein (one protein) interactions described in the
6 Examples. The inventive methods may further be used to investigate drug-protein
7 interactions, nucleic acid-protein interactions, and other chemical molecule-protein
8 interactions.

9 **PROGRAM STORAGE DEVICE**

10 It will be apparent to those of ordinary skill having the benefit of this disclosure
11 that any of the foregoing variations may be implemented by programming one or more
12 suitable general-purpose computers having appropriate hardware. The programming may
13 be accomplished through the use of a program storage device readable by the computer
14 and encoding a program of instructions executable by the computer for performing the
15 operations described above. The program storage device may take the form of, e.g., one
16 or more floppy disks; a CD ROM or other optical disk; a magnetic tape; a read-only
17 memory chip (ROM); and other forms of the kind well-known in the art or subsequently
18 developed. The program of instructions may be "object code," i.e., in binary form that is
19 executable more-or-less directly by the computer; in "source code" that requires
20 compilation or interpretation before execution; or in some intermediate form such as
21 partially compiled code. The precise forms of the program storage device and of the
22 encoding of instructions are immaterial here.

23 **FUNCTION OF PATHWAYS BETWEEN COUPLED POSITIONS**

24

25 The results of the examples set forth above facilitated the mapping of protein
26 energetics. In addition, we have explored the biological roles for the pathways of
27 energetic coupling. We did this by working with large alignments of functionally well-
28 characterized protein families to identify coupled residues through statistical analysis of
29 MSAs, and to determine that these represent the structural elements mediating function

1 both *in vitro* and *in vivo*. We chose two well-known protein families, the p21^{ras} family of
2 GTPases and the hemoglobin family of oxygen carrying proteins, as model systems.
3 Based on the success of our work in identifying coupled residues through statistical
4 analysis, we hypothesized that, for signaling proteins, the prediction of positions for
5 mutagenesis could be achieved because relatively subtle perturbations would disrupt the
6 energetic connectivity and lead to large functional defects *in vivo* due to the uncoupling
7 of signaling events. In other words, we believed that sequence-derived patterns of
8 statistical coupling identified the structural elements of function in protein structure.

9

10 In the p21^{ras} family, we found pathways of statistical connectivity that coupled the
11 guanine nucleotide-binding pocket to the binding site for effector molecules. Our finding
12 was consistent with the fact that this signaling protein family uses the exchange of GDP
13 to GTP nucleotide as a switch for determining binding to effectors. We note that this is a
14 functionally diverse family that shares the GTP switch mechanism as a strategy to
15 regulate many biological processes. Defects in some of these, including p21^{ras}, are
16 associated with many human cancers. For the hemoglobin family, a classic model system
17 for multi-subunit allostery, our statistical analysis using the methods described above
18 revealed pathways of connectivity between pairs of heme groups in the tetrameric protein
19 complex that were exactly consistent with experimentally established principles of
20 oxygen binding allostery. Also, several well-known variants of hemoglobin isolated
21 from human patients that show reduced or absent cooperativity of oxygen binding map to
22 the positions predicted using our statistical analysis.

23

24 Remarkably, the sets of coupled residues in both the p21^{ras} and hemoglobin
25 families formed connected pathways in a state-dependent manner. Residues in the p21^{ras}
26 family coupled to effector binding site positions were only contiguous when the bound
27 nucleotide was GTP, a finding that implied nucleotide-dependent reorganization of
28 thermodynamic connectivity in this protein family. Similarly, the coupled residues in the
29 hemoglobin family were only connected in the de-oxy form (T-state), and demonstrated a
30 discontinuous pattern in the oxygenated form (R-state). This feature was nicely
31 consistent with the observations of Monod, Wyman, and Changeux who in their classic

1 paper on protein allostery, suggested that allosteric ligands mediate “some kind of
2 molecular transition which is induced or stabilized in the protein” (Monod, J. et al., *J.*
3 *Mol. Biol.* 12: 88-118, 1965).

4

5 Based on our work, we suggest that the allosteric molecular transitions represent
6 the relative stabilization of structural states that differ in the pattern of energetic
7 connectivity on the protein, and these differences are the causal basis for the functional
8 switching.

9 **MECHANISMS OF ENERGETIC COUPLING**

10

11 While the present statistical methods are useful in identifying couplings between
12 positions in biological sequences (such as amino-acid positions in protein sequences),
13 they do not by themselves reveal the physical mechanism of the energetic coupling.
14 Nevertheless, the arrangement of coupled residues into ordered pathways through the
15 cores of proteins suggests that the general mechanism of coupling may be simple
16 mechanical compliance of the structure along the coupled pathways. In this view, a
17 structural perturbation at one end of the pathway does not emanate uniformly through a
18 protein; instead, much like fracture lines through many substances, the protein structure
19 accommodates the perturbation along specific directions defined by the pattern of
20 energetic coupling. Thus, much like in hydraulic systems, signals in proteins propagate
21 efficiently and are not locally dissipated during signaling events. If correct, our
22 hypothesis predicts that comparative high-resolution crystal structures of point mutants
23 relative to wild-type protein may reveal pathways of anisotropic structural changes. Our
24 hypothesis further predicts that the overlap in the structural changes of two mutations
25 may reliably map those positions that energetically interact.

26

27 We chose the green fluorescent protein (GFP), a model system well suited for
28 both energetic and structural studies, as an initial test case to develop the necessary
29 structural techniques. Large-scale scanning mutagenesis of GFP revealed hot spots of
30 interaction energy within the chromophore-binding pocket, and double mutant cycles
31 showed specific cases of large and small energetic coupling. To assess the structural

1 correlates of these thermodynamic phenomena, we solved the crystal structures of six
2 GFP proteins representing two complete double mutant cycles and developed an atomic
3 parameter ($\Delta\Delta_{struct}$, described below) that measured the coupled structural change of two
4 perturbations. Specifically, we carried out the analysis of structural coupling for two
5 cases of energetic coupling in GFP: (1) the interaction of mutation at position 145
6 (Y145C) with mutation at position 203 (T203C), and (2) the interaction of protonation of
7 GFP (pH 8.5 to pH 5.5) with mutation at position 203 (T203C). These experiments
8 revealed that (1) single mutations in fact induce structural changes along specific
9 pathways in the protein and (2) energetic couplings quantitatively correlate with well-
10 resolved structural interactions between mutations.

11

12 The principle and one implementation of our method are as follows. A crystal
13 structure of a protein gives four values for each atom in the structure: the three spatial
14 coordinates that give the atom's centroid position in space and one value termed the B-
15 factor, which is related to standard deviation of the atom from its centroid. As used
16 herein, the "centroid" means the center of mass of an atom. A single mutation on a
17 protein may in principle produce structural changes that remain localized to the site of
18 mutation or that may propagate to distant sites. To characterize the effects of a mutation
19 at any given atom, we compared the position and B-factor of the atom in high-resolution
20 crystal structures of the mutant and wild type protein, and calculated the following
21 parameter representing the quantity of change:

22

$$23 \quad \Delta_{struct} = \frac{|\vec{r}_{mut}|}{\sqrt{\sigma_{mut}^2 + \sigma_{wt}^2}},$$

24 where $|\vec{r}_{mut}|$ represents the magnitude of the vector connecting the position of the atom in
25 the mutant structure and the position of the atom in the wild type structure, and σ_{mut} and
26 σ_{wt} represent the standard deviations of the atom in the mutant and wild type structures,
27 respectively. The standard deviations were calculated from the B-factors of each atom as
28 described in Stroud and Fauman (*Protein Science* (1995) 4:2392-2404). This parameter
29 (Δ_{struct}) gave the quantity of structural change for each atom.

1
2 The structural coupling of two mutations is the degree to which the structural
3 change induced by one mutation is different from that induced in the presence of another
4 mutation. To determine this, we solved crystal structures of the wild-type protein, each
5 single mutant protein (mutant 1 and mutant 2), and the double mutant protein. The
6 solving of these crystals structures is well within the skill of one in the art. The following
7 parameter then gave the quantity of structural coupling ($\Delta\Delta_{struct}$) due to the two
8 mutations for each atom:

9

$$10 \quad \Delta\Delta_{struct} = \frac{|\vec{r}_{mut1} - \vec{r}_{mut1|mut2}|}{\sqrt{\sigma_{wt}^2 + \sigma_{mut1}^2 + \sigma_{mut2}^2 + \sigma_{mut1,mut2}^2}},$$

11
12 where \vec{r}_{mut1} represents the vector connecting the position of the atom in the structure of
13 mutant 1 and the position of the atom in the wild type structure, and $\vec{r}_{mut1|mut2}$ represents
14 the vector connecting the position of the atom in the structure of the double mutant
15 (mut1,mut2) and the position of the atom in the structure of mutant 2. Here, σ_{wt}
16 represents the standard deviation of the atom in the wild-type protein; σ_{mut1} represents
17 the standard deviation of the atom in mutant 1; σ_{mut2} represents the standard deviation of
18 the atom in mutant 2; and $\sigma_{mut1,mut2}$ represents the standard deviation of the atom in the
19 double mutant. These standard deviations were calculated from the B-factors of each
20 atom as described in Stroud and Fauman (*Protein Science* (1995) 4:2392-2404).

21
22 Though the perturbation described above comprised mutagenesis, the present
23 methods may be employed for all forms of perturbation. For example, other non-
24 mutagenic perturbations include, but are not limited to, the binding of pharmacological
25 agents, the binding of other proteins, or changes in pH that may alter the protonation of
26 sites in proteins. In addition, it will be understood that as disclosed herein, the source of
27 a perturbation is irrelevant for present purposes. In other words, perturbed biological
28 sequences that exist in nature are as useful as those achieved through human intervention.

1 Human intervention may effect changes through, for example, the binding of
2 pharmacological agents or mutagenesis.

3

4 Our findings may be used to help facilitate the process of optimizing lead
5 compounds during drug design by predicting which positions in a drug binding site act as
6 structurally independent positions, and which act cooperatively with other positions.
7 Such cooperative effects of protein sites may also be the basis for the development of
8 drug resistance. For example, positions that are structurally coupled to drug binding sites
9 represent potential sites for selection of mutations that reduce or eliminate the potency of
10 the drug. The combined usage of our statistical algorithms for sequence analysis together
11 with these crystallographic methods provides a method for prediction of the cooperative
12 interactions at drug binding sites.

13

14 **DNA MICROARRAY ANALYSIS**

15

16 As explained above, the present methods are useful for analyzing non-protein
17 biological sequences. For example, the present methods are useful for analyzing DNA
18 microarray data, where the major current goal is to develop methods to identify the
19 specific interaction of gene products during biological events. Present methods for this
20 analysis typically involve the comparison of genome wide transcriptional changes before
21 and after many perturbations to cells or animals and the clustering of similar patterns of
22 transcriptional change. This work has helped to identify groups of genes that co-vary
23 during many different biological processes and has set the standard for the primary
24 mechanism of discovering relationships between genes.

25

26 An unrealized goal of microarray technology is the ability to map pathways of
27 signaling in cells through the analysis of covariance in gene transcription due to genetic
28 mutation. A single gene knockout shows changes in the expression of tens or hundreds
29 of genes in comparison with wild type suggesting a combination of both local
30 perturbation of a signaling pathway specific to the mutated gene and the propagated
31 effect of the mutation. Also, in many cases the effect is small relative to noise. Prior

1 methods have been unable to map the interaction of the gene of interest in its signaling
2 pathway or identify the changes that are distantly correlated long-range effects of genetic
3 mutation.

4

5 We extended our work to address this problem. Using the publicly available
6 database of microarray data for the yeast mating pathway published by Rosetta
7 Inpharmatics, we determined that the specific pathway of interaction of two gene
8 mutations can be robustly and reliably identified through the non-additivity of their
9 expression profiles.

10

11 The non-additivity of two perturbations in triggering gene expression changes was
12 calculated in the following way. Each perturbation may cause the change in the
13 expression of any other gene in the genome. In this regard, “perturbation” is a broad
14 term, and may include a single gene mutation, multiple gene mutations, an applied
15 pharmacological agent, or a disease state. The quantity of expression change for each
16 gene in the genome due to a single perturbation is given by the fold change in the
17 microarray hybridization signal for that gene. We calculated the coupling of two
18 perturbations as the degree to which the fold change of expression of one gene was
19 different in the presence of a second perturbation. To determine this, we obtained
20 microarray data for four conditions: (a) the unperturbed “wild type” condition, (b)
21 perturbation 1, (c) perturbation 2, and (d) the double perturbation of 1 and 2. The degree
22 of coupling between perturbations 1 and 2 for each gene ($\Delta\Delta E$) is given by:

23

24

$$\Delta\Delta E = kT' \ln\left(\frac{f_1}{f_2}\right),$$

25

26 where f_1 is the fold effect of the gene due to perturbation 1 relative to wild type, and f_2
27 is the fold effect of the gene due to the combined perturbation of 1 and 2 relative to
28 perturbation 2 alone. The calculation of this value for all genes in the genome gives the
29 full analysis of genes responsible for the interaction of two perturbations. Similar to T^*
30 used herein, T' , the “temperature” of the ensemble of this system, is related to the mean
31 transition rates between states, but the energy unit, kT' , in such a system is not

1 necessarily related to that for conventional mechanical systems, or to kT^* described
2 above.

3

4 As in case of protein sites on a sequence alignment, this approach measures the
5 interaction of two genes as the degree to which the expression changes due to mutation in
6 the first are different when tried in the background of mutation in the second.
7 Interestingly, this provides a quantitative measure of the interaction, and provides a list of
8 genes that are responsible for the interaction. In the case of microarray analysis of
9 mutations in the yeast mating pathway data, we were able to extract essentially the entire
10 pathway of the mating factor through analysis of the non-additivity of two mutations
11 (Rst1 and Rst2) in that pathway. In addition, the non-additivity analysis provided signal
12 to noise in distinguishing genes known to be involved in this pathway from those not
13 involved in this pathway.

14

15 All of the methods disclosed and claimed herein can be made and executed
16 without undue experimentation in light of the present disclosure. While the methods of
17 this invention have been described in terms of preferred embodiments, it will be apparent
18 to those of skill in the art that variations may be applied to the methods and in the steps or
19 in the sequence of steps of the methods described herein without departing from the
20 concept, spirit and scope of the invention. All such similar substitutes and modifications
21 apparent to those skilled in the art are deemed to be within the spirit, scope and concept
22 of the invention.

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